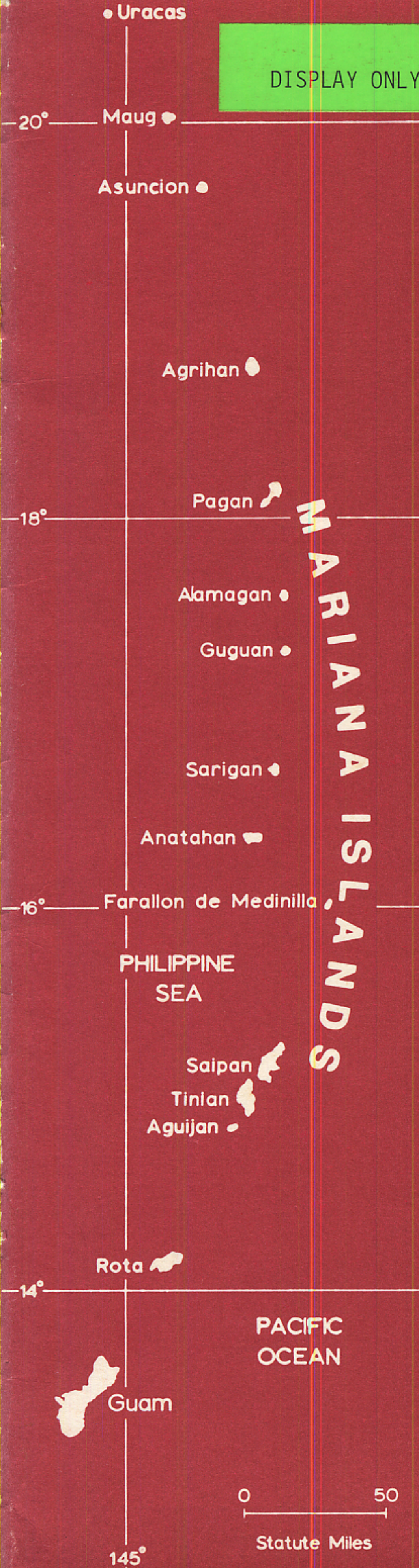

A REVIEW OF TECHNIQUES FOR STUDYING FRESHWATER/SEAWATER RELATIONSHIPS IN COASTAL AND ISLAND GROUNDWATER FLOW SYSTEMS

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UNIVERSITY OF GUAM

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ABSTRACT

This report reviews and summarizes the analytic and numeric methods available for the study of saltwater intrusion into freshwater aquifers in coastal and island situations. The methods are divided into three categories. The first deals with closed-form analytic solutions of steady interface problems using the hodograph method. In these problems, the saltwater and freshwater are separated by an abrupt interface and the location of this interface is determined for simple aquifer geometries and boundary conditions. These solutions can also be used to test the accuracy of numerical techniques. The second category of methods deals with solutions utilizing the Dupuit and Ghyben-Herzberg approximations. The unsteady equations in one and two dimensions that need to be solved are presented. Numerical approximations of these differential equations are suggested and results reported in the literature. The last category of methods deals with numerical solutions of the convection-dispersion equation. A finite element solution of the equations applied to a vertical two-dimensional plane is presented. This method results in the concentration of salt as a function of space and time. The advantages and limitations of all the methods are discussed.

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INTRODUCTION

Of primary interest to communities of oceanic islands is the acquisition of freshwater for domestic and municipal purposes. Because an island is a closed physical system, there is a limited number of methods by which freshwater can be obtained. Such methods include desalinization of seawater, surface catchment of precipitation, and extraction of fresh groundwater. Of the three methods, the extraction of fresh groundwater is generally the most desirable in terms of cost and reliability.

Fresh groundwater beneath a coastal area or oceanic island often occurs as a lens-shaped body bounded by a phreatic surface above and a transition zone below. The overall configuration of the lens is determined by the size, geometry, and permeability of the island; by the extent of urban development, including groundwater extraction; and by the rate of groundwater recharge. Time-related changes in the lens configuration are related to seasonal variations in recharge and variable pumping rates.

The development and management of a freshwater lens is dependent on a firm understanding of the characteristics of lens behavior under a set of current, expected, or proposed conditions. Often physical or mathematical models are employed to predict the response of the groundwater-flow system to various known stresses. From such predictions, present development schemes can be tested; relative merits of various alternatives can be appraised; and long-term groundwater management policies can be formulated. Thus, the model, if properly posed and thoroughly tested, provides valuable insight to the behavior of a groundwater-flow system that otherwise may not be acquired by observations alone and a tool for further development and management of the freshwater resource.

The purpose of this report is to provide a review of various mathematical modeling techniques that apply to the Guam case. Three methods are presented: hodograph analysis, Dupuit and Ghyben-Herzberg approximations, and solution of the convection-dispersion equation in conjunction with the groundwater-flow equations. Each of the three methods can be applied, at least in part, to the groundwater-flow system of northern Guam as well as to other areas of the island underlain by fresh or brackish groundwater.

LITERATURE REVIEW

The most relevant current literature dealing with seawater intrusion and island groundwater-flow systems will be reviewed in the same order as the content of the next three chapters. First, the literature dealing with the exact solution of the steady-state interface using the hodograph plane will be reviewed. Next, the approximate solutions utilizing the Dupuit and Ghyben-Herzberg approximations will be dealt with. Finally, the literature concerning the convection-dispersion equation will be reviewed.

The hodograph method is a technique that was introduced by Hemholtz and Kirchhoff to study discontinuous motion of liquids. The first applications of this method were to free streamline flow and to the calculation of the coefficients of contraction of nozzles and orifices. The method was applied to groundwater flow to determine the location and shape of the phreatic surface as water seeps through an earth dam. Numerous applications to groundwater problems have been illustrated by Polubarinova-Kochina (1962), Aravin (1965), and Harr (1962). Applications to seawater intrusion problems have been described by Henry (1959), Bear (1972), Bear and Dagan (1964), and De Josselin de Jong (1965). This method has provided exact solutions to some problems with simple boundary conditions. As the boundary conditions get to be more complex, the hodograph method may not be able to provide exact solutions, even though numerical and analog techniques may be used to obtain solutions to practical field problems. The exact solutions obtained by the hodograph method may be used to check the accuracy of other numerical methods, for example finite difference and finite element techniques.

Approximate techniques have been developed which make use of the Dupuit and Ghyben-Herzberg approximations. Glover (1958) used the method of complex variables to determine the shape of the steady interface in a confined aquifer. Van der Veer (1977) obtained a non-linear algebraic equation using complex potentials by which he solved for the shape and the location of both the phreatic surface and the interface in an unconfined coastal aquifer with uniform vertical recharge. Numerical solutions of one-dimensional, time-varying flow have been obtained by Anderson (1976) as applied to strip oceanic islands. Fetter (1971) presented a two-dimensional, steady-state model to study water-table elevations for an oceanic island of any shape. An unsteady model in two-dimensions has been developed by Ayers (1979) and applied to the aquifers in Bermuda.

All the models referred to previously have made the assumption that a sharp interface exists between the fresh and seawater. In many cases this is a reasonable assumption and, when this can be confirmed by field data, models similar to those described above can be used confidently for aquifer management and future planning. If, however, field data indicate that the transition zone is extensive, then the convection-dispersion equation for salt transport has to be solved in addition to the groundwater flow equations. The first attempt at solving these equations was reported by Pinder and Cooper (1970). They solved the flow equations by the alternate-direction-implicit (ADI) finite difference procedure and solved the salt transport equation using the method of characteristics. A similar problem was treated by Segol, Pinder and Gray (1975) using the finite element technique, including the possibility of a layered or otherwise non-homogeneous porous medium. A similar finite element program was developed for application to the aquifers of eastern Virginia by Desai and Contractor (1976) and Wu, Desai and Contractor (1976).

SOLUTIONS OF THE STEADY-STATE INTERFACE USING THE HODOGRAPH METHOD

The hodograph plane is a two-dimensional plot of the horizontal and vertical velocities in a groundwater flow domain. The flow domain can be drawn in the (x,y) plane and at every point in the flow domain the horizontal and vertical velocities (u,v) can be determined and a point plotted in the (u,v) plane. Every point in the flow domain (x,y) can thus be mapped into the hodograph (u,v) plane. If the boundaries in the flow domain are transformed into corresponding boundaries in the hodograph plane, then the interior of the flow domain will map into corresponding points within the boundaries in the hodograph plane.

Let $z=x+iy$ and $W=u+iv$, where $i = \sqrt{-1}$. Also, let $w = f(z) = \phi(x,y) + i\psi(x,y)$. When w is an analytic (regular or holomorphic) function of z, then $\phi(x,y)$ can be shown to be the velocity potential and $\psi(x,y)$ can be shown to be the stream function. Every analytic function of z thus forms a possible fluid flow pattern that can be described by equipotential lines and streamlines. The derivative of w with respect to z can be shown to have a real part equal to the horizontal velocity u and an imaginary part that is equal to the negative of the vertical velocity v; i.e., $\frac{dw}{dz} = W' = u - iv$. The hodograph plane is the reflection of the W' plane about its horizontal (u) axis.

The transforms of the more commonly-encountered boundaries in the z plane into the hodograph (W) plane follow:

1) Impervious Boundary.

In the z plane, an impervious boundary is specified by $\frac{\partial\phi}{\partial n} = 0$ or ψ = constant, since the flow is tangent to it. If the impervious boundary in the z plane is a straight line, then the equation of the corresponding line in the hodograph plane is $v = u \tan \alpha$, where α is the angle the imprevious boundary makes with the +x axis. Thus, the transform is a straight line through the origin and parallel to the impervious boundary.

2) Reservoir Boundary

This boundary would be specified in the z plane by ϕ = constant or $\frac{\partial\psi}{\partial s} = 0$. If the reservoir boundary is a straight line, then the transform becomes $v = u \cot \alpha$, where α is the inclination of the reservoir boundary with the +x axis. This transform is a line going through the origin perpendicular to the reservoir boundary.

3) Phreatic Surface.

a) With Accretion.

The shape of the phreatic surface is unknown 'a priori' and is, in general, curvilinear. Since the pressure along this boundary is atmospheric ($p = 0$), the boundary condition is $\phi = Ky$. The corresponding boundary condition in the hodograph plane is given by $u^2 + (v + K/2)^2 = (K/2)^2$. This is the equation of a circle with center $(0, -K/2)$ and radius of $K/2$. K is the permeability of the porous medium.

b) With Accretion

If infiltration occurs at a flow rate per unit area of N, then the phreatic surface is no longer a streamline and $\psi = Nx + \text{constant}$. The corresponding equation in the hodograph plane becomes $u^2 + [v + (K - N)/2]^2 = [(K + N)/2]^2$. This is the equation of a circle with center at $[0, -(K - N)/2]$ and radius $(K + N)/2$.

4) Surface of Seepage.

The surface of seepage is not a streamline and, since it is at atmospheric pressure ($p = 0$), the boundary condition $\phi = Ky$ applies. Let the surface of seepage be a straight line inclined at an angle with the +x axis. The transform of this boundary in the hodograph plane is given by $v = u \cot \alpha - K$; i.e., by a straight line through $(0, -K)$ and perpendicular to the surface of seepage.

5) Interface Between Two Immiscible Fluids of Different Densities.

The assumption is made that the heavier liquid is stationary. γ_s denotes its specific weight and γ_f denotes the specific weight of the lighter fluid. The boundary condition that the pressure is continuous across the interface gives rise to the equation $\phi = \left(\frac{\gamma_s - \gamma_f}{\gamma_f}\right) Ky$, where $\gamma_s > \gamma_f$. This boundary condition is very similar to the boundary condition for a phreatic surface without accretion. The interface is also a streamline. The transform of the interface in the hodograph plane is $u^2 + (v - K'/2)^2 = (K'/2)^2$, where $K' = K(\gamma_s - \gamma_f)/\gamma_f$. This is the equation of a circle with center at $(0, K'/2)$ and radius $K'/2$.

6) Surface of Seepage of Lighter Liquid (γ_f) into a Reservoir of Stationary Heavier Liquid (γ_s).

To maintain continuity of pressure across the surface of seepage, $\phi = \left(\frac{\gamma_s - \gamma_f}{\gamma_f}\right) Ky$. However, in this case, the boundary is not a streamline. The transform of the boundary in the hodograph plane is $v = u \tan(\alpha - 90) + K'$. Thus, the transform is a straight line going through K' and perpendicular to the surface of seepage.

Application of the Hodograph to the Interface in a Confined Coastal Aquifer

Figure 1(a) shows a diagram of a confined coastal aquifer in which there is a flow (Q) of freshwater in an aquifer with permeability K. The boundaries of the flow domain can be transformed into the hodograph (W) plane as shown in Figure 1(b). The complex potential (w) plane is shown in Figure 1(c). The hodograph (W) plane is reflected about the u axis, resulting in the W' plane. Since the hodograph plane has a semi-circle as one of its boundaries, the boundaries are plotted in a new plane (W*) that is the reciprocal of W'; i.e., $W^* = K'/W'$. Figure 1(d) shows that all the boundaries in this (W*) plane are straight lines. The figures in 1(c) and 1(d) are transformed into the upper half of the 't' plane by means of the Schwarz-Christoffel transformation. These transformations are given below:

$$t = \cosh(\pi W^*),$$

$$w + \frac{Q}{\pi} \cosh^{-1}[(at-1)/(a-t)]$$

where:

$$a = \cosh(\pi K'D/Q) = \cosh(\pi K'/q_o).$$

Since:

$$W' = \frac{dw}{dz} = K' / W^*,$$

$$dz = \frac{W^*}{K'}\, dw = \frac{W^*}{K'}\,\frac{dw}{dt}\, dt$$

$$= \frac{Q\sqrt{a^2 - 1}\,\cosh^{-1}(t)}{\pi^2 K'(a-t)\sqrt{t^2 - 1}}\, dt.$$

To derive the parametric equations for the interface, the differential equation shown above needs to be intergrated from $t = -\infty$ to $t = -1$.

Let $\eta = \cosh^{-1}(-t)$. Therefore:

$$dz = \frac{Q\sqrt{a^2-1}}{\pi^2 K'(a + \cosh\eta)}(\eta + i\pi)\, d\eta.$$

Separating the real and imaginary part of the equation, one obtains:

$$dx = \frac{Q\sqrt{a^2-1}}{\pi^2 K'(a + \cosh\nu)}\,\nu d\nu$$

$$dy = \frac{Q\sqrt{a^2-1}}{\pi K'(a + \cosh\nu)}\, d\nu.$$

Integrating the latter equation, one obtains:

$$y = \frac{Q}{\pi K'}\, F_1(\nu)$$

where:

$$F_1(\nu) = \ln\frac{a + 1 + \sqrt{a^2-1}\tanh(\nu/2)}{a + 1 - \sqrt{a^2-1}\tanh(\nu/2)}.$$

For large values of a, y may be approximated by:

$$y = \frac{Q}{\pi K'} [\nu - F_3(\nu)]$$

where:

$$F_3(\nu) = \ln \frac{e^{\nu} + 2a}{1 + 2a}.$$

Integrating the equation for dx, one obtains:

$$x = \frac{Q}{\pi^2 K'} F_2 (\nu)$$

where:

$$F_2(\nu) = \nu F_1(\nu) - \int_0^{\nu} F_1(\nu) d\nu.$$

The values of the functions can be obtained by numerical integration. A graphical solution for the interface shape and location is presented in Bear (1972) and Bear and Dagan (1964). This solution can be used to check the accuracy of new numerical methods. Other solutions for this problem have been compared with this solution, Glover (1958), and found to be accurate enough for engineering purposes. It can also be shown that the Dupuit and Ghyben-Herzberg approximations can be used to simplify the solution technique without much loss of accuracy.

SOLUTIONS UTILIZING THE DUPUIT AND GHYBEN - HERZBERG APPROXIMATIONS

Glover (1958) determined the shape of the seawater interface in an infinite aquifer. He utilized Kozeny's analysis of free surface flow towards a horizontal drain. Glover showed that the interface has a parabolic shape given by the equation:

$$y^2 - \frac{2Q}{\gamma K} x - \frac{Q^2}{\gamma^2 K^2} = 0$$

Van der Veer (1977) analyzed the position of the seawater interface in an infinite medium with a phreatic surface and with infiltration from precipitation. For the one-dimensional case, he showed that:

$$H^2 = \frac{\frac{C_2}{K} - \frac{2C_1}{K}x - \frac{N}{K}x^2}{\gamma'(\gamma'+1)}$$

where $\gamma' = (\gamma_s - \gamma_f)/\gamma_f$ and C_1 and C_2 are integration constants. H is the depth of interface below sea level. $h = \gamma' H$ = height of phreatic surface above sea level and $q = Nx + C_1$.

For the two-dimensional case, he presented the following equations:

$$H^2 = \frac{-\left\{\frac{N x^2}{K} + \frac{2q^*x}{K}\right\}}{(\gamma'+1)(\gamma'+\frac{N}{K})}$$

$$h^2 = -\left(\frac{Nx^2}{K} + \frac{2q^*x}{K}\right)\frac{\left(\gamma'+\frac{N}{K}\right)}{(\gamma'+1)} - \left(\frac{q^*}{K}\right)^2 \frac{\left[1-\left(\gamma'+\frac{N}{K}\right)\right]}{(\gamma'+1)\left(1-\frac{N}{K}\right)}$$

$$\ell_e = \frac{q^*}{N}\left\{1 - \left[1 - \left(\frac{N}{K}\right)\frac{\left(1-\left(\gamma'+\frac{N}{K}\right)\right)}{\left(1-\frac{N}{K}\left(\gamma'+\frac{N}{K}\right)\right)}\right]^{\frac{1}{2}}\right\}$$

ℓ_e = distance of shoreline to intersection of interface with sea level.

where $q^* = q + N\ell_e$.

Anderson (1976) reported on a numerical a model for unsteady, one-dimensional flow beneath strip oceanic islands. The delayed interface response (DIR) model that she worked with solved the following partial differential equation:

$$\frac{\partial}{\partial x}\left[\bar{K}\ (h + H)\ \frac{\partial h}{\partial x}\right] = S\ \frac{\partial h}{\partial t} - N\ (x,t)$$

where $H = \gamma' h$ and S = the storage coeficient.

Anderson solved this equation using the predictor-corrector technique described by Douglas and Jones (1963) and the Thomas algorithm for solving tri-diagonal matrices [Remson et al (1971)]. She applied his model to the groundwater situation on the South Fork of Long Island, N. Y. The water level in one of the wells was simulated satisfactorily for a year by varying the recharge rates.

Fetter (1971, 1972) presented a steady-state, two-dimensional model that he applied to the South Fork of Long Island, N. Y. Recharge to the aquifer was determined from the precipitation, evapotranspiration, gaged streamflow and consumptive use of water. He compared the computed elevation of the interface with the measured elevation of the top of the zone of diffusion in a number of wells. Except for one well, the computed elevations were within 6% of the measured elevations.

A time-dependent, two-dimensional model was developed for two aquifers in Bermuda by Ayers (1979). This study seems to be the most comprehensive work involving a sharp interface to date He used the two-dimensional extension of Anderson's delayed interface response (DIR) model:

$$\frac{\partial^2(h^2)}{\partial x^2} + \frac{\partial^2(h^2)}{\partial y^2} = \frac{2\ Sy}{K\ (1 + \gamma')}\ \frac{\partial h}{\partial t} - \frac{2\ N(x,y,t)}{K\ (1 + \gamma')}$$

By substituting $\frac{\partial h}{\partial t} = \frac{1}{2h} \frac{\partial h^2}{\partial t}$ and $h^* = h^2$, the equation becomes:

$$\frac{\partial^2 h^*}{\partial x^2} + \frac{\partial^2 h^*}{\partial y^2} = \frac{Sy}{Kh\ (1 + \gamma')} \frac{\partial h^*}{\partial t} - \frac{2\ N\ (x,y,t)}{K\ (1 + \gamma')}$$

The time-derivative term is non-linear because of the presence of h in the denominator. It can be linearized by using an average value $\bar{h}$ for h. Thus, the linearized equation becomes:

$$\frac{\partial^2 h^*}{\partial x^2} + \frac{\partial^2 h^*}{\partial y^2} = \frac{Sy}{K\ (1 + \gamma')\ \bar{h}} \frac{\partial h^*}{\partial t} - \frac{2\ N(x,y,t)}{K\ (1 + \gamma')}.$$

The finite - difference equivalent of the above equation is next written using:

i as a spatial index in the x direction

j as a spatial index in the y direction

n as an index of time and

m as a counter of the iteration number.

Ayers used the iterative alternate direction implicit (IADI) method proposed by Peaceman and Rachford (1955). The iterative method was preferred over the non-iterative method because the convergence error can be specified and because larger time steps can be used [Trescott et al (1976)]. Each of the two finite difference equations was solved iteratively using the tri-diagonal Thomas algorithm [Remson et al (1971)]:

$$\frac{h^*_{i+1,j,n+1,m} - 2h^*_{i,j,n+1,m} + h^*_{i-1,j,n+1,m}}{\Delta x^2}$$

$$+ \frac{h^*_{i,j+1,n+1,m+1} - 2h^*_{i,j,n+1,m+1} + h^*_{i,j-1,n+1,m+1}}{\Delta y^2}$$

$$= \frac{S_{y\ i,j}}{K_{i,j}\ (1+\gamma')\bar{h}_{i,j,n}} \quad \frac{h^*_{i,j,n+1,m+1} - h^*_{i,j,n,m+1}}{\Delta t} - \frac{2N_{i,j,n+1}}{K_{i,j}(1+\gamma')}$$

$$\frac{h^*_{i+1,j,n+1,m+2} - 2h^*_{i,j,n+1,m+2} + h^*_{i-1,j,n+1,m+2}}{\Delta x^2}$$

$$+ \frac{h^*_{i,j+1,n+1,m+1} - 2h^*_{i,j,n+1,m+1} + h^*_{i,j-1,n+1,m+1}}{\Delta y^2}$$

$$= \frac{S_{y,i,j}}{K_{i,j}(1+\gamma')\ \bar{h}_{i,j,n}} \quad \frac{h^*_{i,j,n+1,m+2} - h^*_{i,j,n,m+2}}{\Delta t} - \frac{2N_{i,j,n+1}}{K_{i,j}(1+\gamma')}$$

The equations shown above were applied to all the interior nodes of the island grid. The head h* at the boundary nodes was specified by the boundary condition $h^*_{i,j} = 0$ for all n and m. Initial conditions were specified as $h^*_{i,j,0} = h^*_{o_{i,j}}$ for all nodes. The model was used to simulate steady- and unsteady-state water-table elevations which were compared to 1976 average prototype water levels at six observation wells on Somerset Island. Unsteady simulation results of water elevations in response to varying monthly recharge and pumping schedules were compared to prototype data at 36 observation wells penetrating Devonshire Lens (Main Island). The comparison was good except at wells close to the shoreline where the Dupuit assumptions are violated.

SOLUTIONS OF THE CONVECTION-DISPERSION EQUATION

The important details of the finite element approach to the solution of the convection-dispersion equation are described below. For a vertical (x,y) section, the equations that need to be solved are:

$$\varepsilon \frac{\partial}{\partial x}(\rho u) + \varepsilon \frac{\partial}{\partial y}(\rho v) - \rho' W_s = 0$$

$$u + \frac{K_{11}}{\varepsilon\ \mu} \frac{\partial p}{\partial x} = 0$$

$$v + \frac{K_{22}}{\varepsilon\ \mu} \left(\frac{\partial p}{\partial x} + g\right) = 0$$

$$\frac{\partial}{\partial x}\left(D_{11} \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{22} \frac{\partial c}{\partial y}\right) - \frac{\partial (uc)}{\partial x} - \frac{\partial (vc)}{\partial y} - \frac{c' W_s}{\varepsilon} - \frac{\partial c}{\partial t} = 0$$

$$\rho = \rho_w + Ec.$$

where:

ε = Porosity

ρ = Mass density of fluid

ρ' = Mass Density of recharge liquid

K_{11} = Intrinsic permeability in the x direction

K_{22} = Intrinsic permeability in the y direction

μ = Coeficient of viscosity

W_s = Well recharge in volume flow rate/unit area (dx·dy)

D_{11} = Dispersion coeficient (L^2/T) in the x direction

D_{22} = Disperison coeficient (L^2/T) in the y direction

c' = Concentration of recharge liquid.

c = Concentration of aquifer liquid

ρ_w = Mass density of fresh water

E = A constant = 0.7 for salt concentration

u = Seepage velocity in the x direction

v = Seepage velocity in the y direction

The first equation is related to the conservation of total fluid mass, including fluid mass due to aquifer recharge and pumping. The second equation is Darcy's law applied in the horizontal direction. The third equation is Darcy's law applied in the vertical direction. If Dupuit's assumption was going to be used, then this equation would not be necessary. The fourth equation is the convection - dispersion equation which has to be used if the Ghyben - Herzberg assumption is not held valid. The fifth equation relates the concentration of the salt water to the density of the fluid.

Even though all the dependent variables p,u,v,c, and ρ are assumed to be time dependent, only the fourth equation has a time dependent term in it. This is because it has been assumed that pressure propagates more rapidly in the system than salt transport. It should also be noted that the velocities u and v are calculated simultaneously with p, so as to ensure that the velocities will be continuous between elements. Continuity of velocities between elements is one of the reasons why numerical stability is maintained at high Peclet (VL/D) numbers. In the fourth equation, if the first two diffusion terms dominate over the next two convective terms, then the numerical scheme provides satisfactory results. If, on the other hand, the convective terms are dominant, then in the region of high pressure gradients, numerical difficulties have been known to lead to negative values of concentration. Thus, the formulation must be such that the velocity field is properly represented.

The procedure for solving the five equations is as follows: The first three equations are solved for a first estimate of the pressure and velocities. The velocities are then used in the fourth equation to solve for the concentrations. The concentrations are used to update the density distribution using the fifth equation. The first three equations are then solved again using the new density distribution and the cycle repeated till convergence is reached.

The Galerkin weighted residual method is used to obtain numerical solutions of the partial differential equations. The principal steps in this method will be outlined. First, the variation of the dependent variables is expressed in terms of the values of the variables at the 'n' nodes of an element as follows:

$$p(x,y,t) = \sum_{j=1}^{n} N_j(x,y)\ p_j(t)$$

$$u(x,y,t) = \sum_{j=1}^{n} N_j(x,y)\ u_j(t)$$

$$v(x,y,t) = \sum_{j=1}^{n} N_j(x,y)\ v_j(t)$$

$$c(x,y,t) = \sum_{j=1}^{n} N_j(x,y)\ c_j(t)$$

where $N_j(x,y)$ are the basis of interpolating functions and p_j, u_j, v_j, and c_j are the time dependent values at the nodes of the element.

If approximate values of the variables at the nodes are assumed and these approximations substituted into the differential equations, the right hand side of the equations will not be equal to zero but will be equal to some residue R. More accurate values of the variables can be found if the weighted residues are reduced to zero. Let the residues of the four differential equations be R_p, R_u, R_v, and R_c. In the Galerkin method, these residues are weighted by the basis functions, integrated over the element area and set equal to zero:

$$\int_A R_p(u,v)\ N_j(x,y)\ dA = 0$$

$$\int_A R_u(u,p)\ N_j(x,y)\ dA = 0$$

$$\int_A R_v(v,p)\ N_j(x,y)\ dA = 0$$

$$\int_A R_c(u,v,c)\ N_j(x,y)\ dA = 0$$

Substituting the differential equations and the variable approximations into the equations above, one obtains:

$$\int_A \left[\left(\varepsilon \frac{\partial}{\partial x}\left(\rho \sum_{j=1}^{n} N_j u_j\right) + \varepsilon \frac{\partial}{\partial y}\left(\rho \sum_{j=1}^{n} N_j v_j\right) - \rho' W_s\right]\ N_i\ dA = 0$$

$$\int_A \left[\left(\sum_{j=1}^{n} N_j u_j + \frac{K_{11}}{\varepsilon\mu}\ \left(\frac{\partial}{\partial x}\left[\sum_{j=1}^{n} N_j p_j\right]\right)\right]\ N_i\ dA = 0$$

$$\int_A \left(\sum_{j=1}^{n} N_j v_j + \frac{K_{22}}{\varepsilon\mu} \left(\frac{\partial}{\partial y} [\sum_{j=1}^{n} N_j p_j] + g \right) \right) N_i \, dA = 0$$

$$\int_A \left[\frac{\partial}{\partial x} \left(D_{11} \frac{\partial}{\partial x} [\sum_{j=1}^{n} N_j c_j] \right) + \frac{\partial}{\partial y} \left(D_{22} \frac{\partial}{\partial y} [\sum_{j=1}^{n} N_j c_j] \right) \right.$$

$$- \frac{\partial}{\partial x} \left([\sum_{j=1}^{n} N_j u_j] [\sum_{j=1}^{n} N_j c_j] \right) - \frac{\partial}{\partial y} \left([\sum_{j=1}^{n} N_j v_j] [\sum_{j=1}^{n} N_j c_j] \right)$$

$$\left. - \frac{c' W_s}{\varepsilon} - \frac{\partial}{\partial t} [\sum_{j=1}^{n} N_j c_j] \right) N_i \, dA = 0$$

The first three equations can be expanded and put in the following matrix form:

$$\begin{bmatrix} H \end{bmatrix} \begin{pmatrix} p \\ u \\ v \end{pmatrix} + \begin{bmatrix} F \end{bmatrix} = 0$$

where:

$$H_{ij} = \int_A \left[\begin{array}{c|c|c} 0 & \varepsilon \frac{\partial}{\partial x} (\rho N_j) N_i & \varepsilon \frac{\partial}{\partial y} (\rho N_j) N_i \\ \hline \frac{K_{11}}{\varepsilon\mu} \frac{\partial N_j}{\partial x} N_i & N_j N_i & 0 \\ \hline \frac{K_{22}}{\varepsilon\mu} \frac{\partial N_j}{\partial y} N_i & 0 & N_j N_i \end{array} \right] dA$$

and:

$$F_i = \int_A \begin{bmatrix} -\rho' W_s N_i \\ \\ 0 \\ \\ \frac{K_{22}}{\varepsilon\mu} \rho g N_i \end{bmatrix} dA$$

The convection-dispersion equation can be written in the following form.

$$\int_A \left\{ \sum_{j=1}^{n} \left[\frac{\partial}{\partial x} \left(D_{11} \frac{\partial N_j}{\partial x} \right) N_i c_j + \frac{\partial}{\partial y} \left(D_{22} \frac{\partial N_j}{\partial y} \right) N_i c_j \right. \right.$$

$$- \frac{\partial}{\partial x} \left([N_j u_j] \; [N_j c_j] \right) N_i - \frac{\partial}{\partial y} \left([N_j v_j] \; [N_j c_j] \right) N_i$$

$$\left. \left. - \frac{c' W_s}{\varepsilon} N_i - N_j N_i \frac{\partial c}{\partial t} \right] \right\} dA = 0$$

The first two terms can be expanded using Green's Theorem.

$$\int_A \left[N_i \frac{\partial}{\partial x} \left(D_{11} \frac{\partial N_j}{\partial x} \right) + N_i \frac{\partial}{\partial y} \left(D_{22} \frac{\partial N_j}{\partial y} \right) \right] dx\, dy$$

$$= - \int_A \left(D_{11} \frac{\partial N_j}{\partial x} \frac{\partial N_i}{\partial x} + D_{22} \frac{\partial N_j}{\partial y} \frac{\partial N_i}{\partial x} \right) dx\, dy$$

$$+ \int_c \left(D_{11} \frac{\partial N_j}{\partial x} N_i \, dy + D_{22} \frac{\partial N_j}{\partial y} N_i \, dx \right)$$

Substituting back into the previous equation and expressing the final equation in matrix form, one obtains:

$$\begin{bmatrix} L \end{bmatrix} \begin{Bmatrix} c \end{Bmatrix} + \begin{bmatrix} M \end{bmatrix} \begin{Bmatrix} \dot{c} \end{Bmatrix} + \begin{Bmatrix} R \end{Bmatrix} = 0$$

where:

$$L_{ij} = \int_A \left(D_{\alpha\beta} \frac{\partial N_j}{\partial x_\alpha} \frac{\partial N_i}{\partial x_\beta} + \frac{\partial}{\partial x_\alpha} (\Sigma N_j v_\alpha) N_j N_i + \frac{\partial N_j}{\partial x_\alpha} v_\alpha N_i \right) dA$$

$$M_{ij} = \int_A N_j N_i \, dA$$

$$R_i = \int_A \frac{c' W_s}{\varepsilon} N_i \, dD - \int_c D_{\alpha\beta} N_i \frac{\partial}{\partial x_\beta} (\Sigma c_k N_k) \ell_\alpha \, d\ell$$

The time derivative $\dot{c} = \frac{\partial c}{\partial t}$ is written in fintie-difference form as $\frac{1}{\Delta t} \left(c\big|_{t+\Delta t} - c\big|_t \right)$ and, hence, the final equation becomes:

$$\left(\begin{bmatrix} L \end{bmatrix} + \frac{1}{\Delta t} \begin{bmatrix} M \end{bmatrix} \right) \begin{Bmatrix} c \end{Bmatrix}_{t+\Delta t} = \frac{1}{\Delta t} \begin{bmatrix} M \end{bmatrix} \begin{Bmatrix} c \end{Bmatrix}_t - \begin{Bmatrix} R \end{Bmatrix}$$

The finite element technique allows one to use different types of elements and different types of basis functions. Thus, Lee and Chen (1974) have used triangular elements and linear and quadrilaterals have been used by Segol et al (1975), Desai and Contractor (1976), and Wu et al (1976). Results of programs with different elements have produced similar results when compared with other analytical results.

SUMMARY AND DISCUSSION

This report has summarized some of the techniques used in the investigations of freshwater/seawater relationships common to groundwater-flow systems of oceanic islands. The choice of the technique that is appropriate for a given aquifer depends on several factors, including the data available to define the geometry and hydrogeology of the aquifer, the output expected of the model, the data available to calibrate and verify the model, and the economic worth of the predicted response to the managers of the aquifer. The technique of analysis should also be tailored to the peculiarities of the aquifer under consideration. No one model has been found to be best for all aquifers and all requirements. Thus, some aquifers may not extend much in the vertical direction but may be very extensive in the horizontal direction and other aquifers may resemble strip oceanic islands. Existing data from observation wells may indicate the interface to be sharp or diffused. All of these points need to be considered in the selection of the model that is appropriate for a given aquifer.

The numerical procedure used in a given model should also be adopted after taking into account several considerations. The memory capacity and execution speed of the computer may control the number of nodes one can use in the model. Finite difference methods have been developed for a wide variety of problems and their convergence and stability characteristics studied for linearized systems. The finite element technique has several advantages of its own such as the ability to choose different types of elements and basis functions and an easier way to handle certain types of boundary conditions. If the overall geometry of the aquifer is narrow and long, then it would help to have elements that are narrow and long. The finite element technique allows one to select the shape and size of the elements to conform to some other criterion.

Several numerical techniques have been developed to solve large numbers of simultaneous equations. A technique that is efficient in memory requirement and computational speed should be selected. These techniques range all the way from Gauss elimination schemes, solution of banded matrices, solution of sparse matrices, to the latest frontal methods. It is essential that the solution technique chosen be matched to the needs of the problem.

A numerical model which simulates the groundwater-flow conditions beneath an island can be a very useful tool in managing the aquifer. The model must be developed in consulation with all personnel likely to provide data input and the model must be explained to and understood by those individuals managing the aquifer on a continuing basis.

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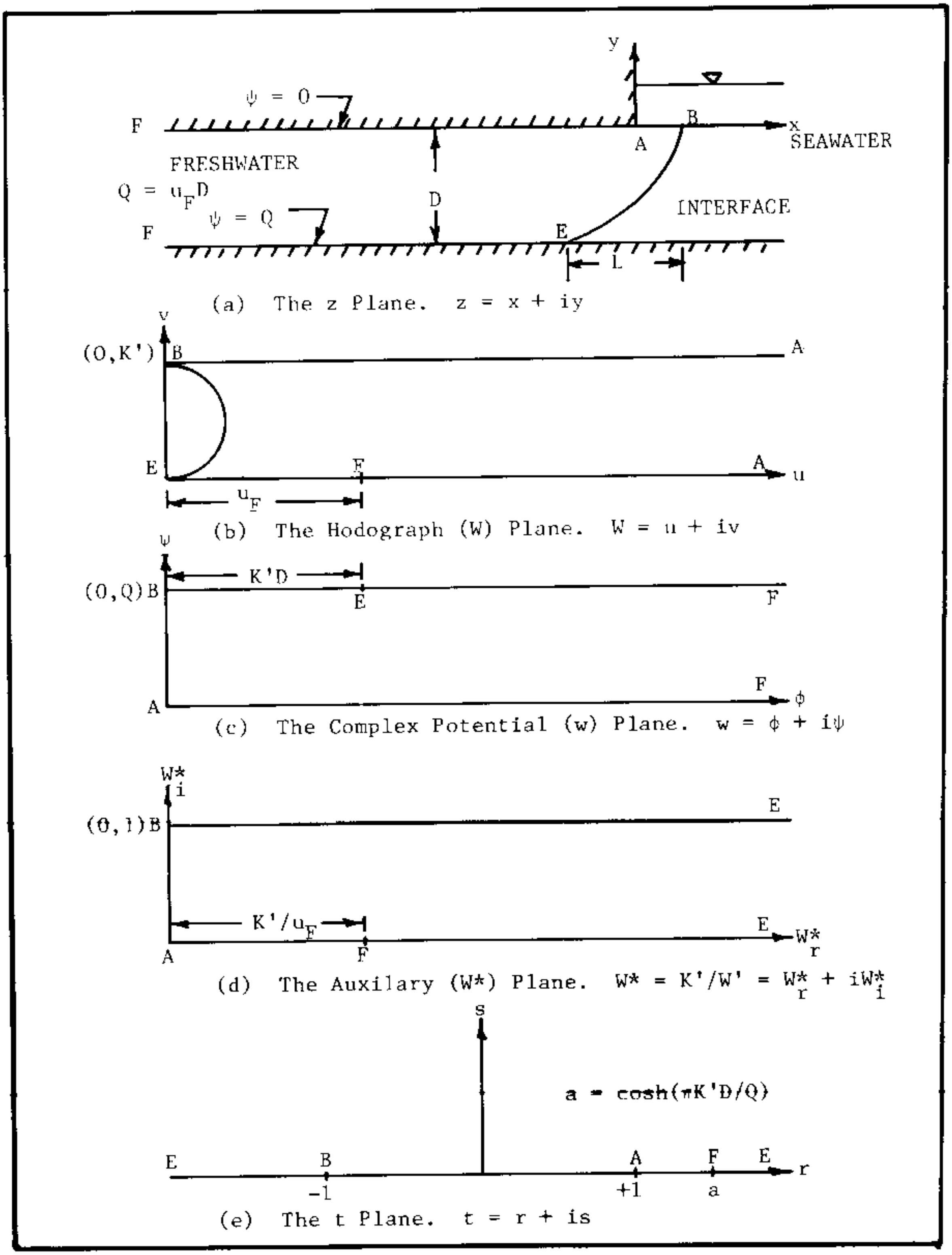

Figure 1. Transformations for the Solution of the Seawater Interface in a Confined Coastal Aquifer Using the Hodograph.